

Flavorful Z' signatures at LHC and ILC

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Abstract

There are lots of new physics models which predict an extra neutral gauge boson, referred as Z' -boson. In a certain class of these new physics models, the Z' -boson has flavor-dependent couplings with the fermions in the Standard Model (SM). Based on a simple model in which couplings of the SM fermions in the third generation with the Z' -boson are different from those of the corresponding fermions in the first two generations, we study the signatures of Z' -boson at the Large Hadron Collider (LHC) and the International Linear Collider (ILC). We show that at the LHC, the Z' -boson with mass around 1 TeV can be produced through the Drell-Yan processes and its dilepton decay modes provide us clean signatures not only for the resonant production of Z' -boson but also for flavor-dependences of the production cross sections. We also study fermion pair productions at the ILC involving the virtual Z' -boson exchange. Even though the center-of-energy of the ILC is much lower than a Z' -boson mass, the angular distributions and the forward-backward asymmetries of fermion pair productions show not only sizable deviations from the SM predictions but also significant flavor-dependences.

The search for new physics beyond the Standard Model (SM) is one of the most important issues of particle physics today. In a class of new physics models, the SM gauge group is embedded in a larger gauge group and such a model often predicts an electrically neutral massive gauge boson, referred as Z' -boson, associated with the original gauge symmetry breaking into the SM one. There are many example models such as the left-right symmetric model [1], Grand Unified Theories based on the gauge groups $SO(10)$ [2] and E_6 [3], and string inspired models [4] (for a review, see, for example, [5]).

It will be very interesting if a Z' -boson is discovered at future collider experiments such as the LHC and ILC. Current limits for the direct production at the Tevatron and indirect effects from LEP experiments imply that the Z' -boson is rather heavy and has a very small mixing with the SM Z -boson. No evidence of a signal has been found, and the lower limits on Z' mass at 95% confidence level are set to be in the range from 650 to 900 GeV, depending on the considered theoretical models [6].

Recently studies about measurement of the Z' -boson at the LHC have been performed [7]. Through the Drell-Yan process, $pp \rightarrow Z'X \rightarrow \ell^+\ell^-X$, a Z' -boson could be discovered at the LHC if its mass lies around TeV scale with typical electroweak scale couplings to the SM fermions. Once a Z' -boson resonance is observed at the LHC, the Z' -boson mass can be precisely measured. The next task is to precisely measure other properties of the Z' -boson, such as couplings to the SM particles, its spin, etc. Future e^+e^- linear colliders, such as the ILC, will be capable of such a task, even if the collider energy is not sufficiently high to produce the Z' -boson. For example, the precision goal of the ILC can allow us to indicate the existence of Z' -boson with mass up to 6 times of center-of-mass energies of the collider [8].

In general, the coupling of Z' -boson with the SM fermions can be flavor-dependent. In fact, such a class of models has been proposed by many authors [9, 10, 11, 12]. If this is the case, the signature of Z' -boson should show flavor-dependences and the collider phenomenology of Z' -boson would be more interesting. In this Letter, we take a simple model recently proposed [12], where the SM fermions in the third generation have couplings with the Z' -boson different from those of the corresponding fermions in the first two generations, and study (flavor-dependent) Z' -boson signatures at the LHC and ILC.

Let us first give a brief review on a recently proposed “top hypercharge” model [12]. This model is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ and the SM fermions in the first two generations have hypercharges only under $U(1)_1$ while the third generation fermions have charges only under $U(1)_2$. A complex scalar field, Σ , in the representation $(\mathbf{1}, \mathbf{1}, \mathbf{1}, 1/2, -1/2)$ is introduced, by whose vacuum expectation value (VEV) ($\langle \Sigma \rangle = u/\sqrt{2}$) the gauge symmetry $U(1)_1 \times U(1)_2$ is broken down to the SM $U(1)_Y$. Associated with this gauge symmetry breaking, the mass eigenstates of two gauge bosons are described as

$$\begin{pmatrix} B_\mu \\ \tilde{B}_\mu \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} B_\mu^1 \\ B_\mu^2 \end{pmatrix}, \quad (1)$$

where $B_\mu^{1,2}$ are the gauge boson of $U(1)_{1,2}$, and the mixing angle ϕ is defined by $\tan \phi = g'_1/g'_2$, the ratio between the coupling constants of $U(1)_{1,2}$. The corresponding masses are $m_{B_\mu}^2 = 0$ and $m_{\tilde{B}_\mu}^2 = (g_1'^2 + g_2'^2)u^2/4$, and the massless state B_μ is nothing but the SM $U(1)_Y$ gauge field.

In terms of B_μ and \tilde{B}_μ , the covariant derivative with respect to $SU(2)_L \times U(1)_1 \times U(1)_2$ for a fermion with a charge (Y_1, Y_2) under $U(1)_1 \times U(1)_2$ is given as

$$D_\mu^f = \partial_\mu - igW_\mu^a T^a - ig'Y B_\mu - ig'(-Y_1 \tan \phi + Y_2 \cot \phi) \tilde{B}_\mu, \quad (2)$$

where $Y = Y_1 + Y_2$ is a hypercharge under the $U(1)_Y$, and the $U(1)_Y$ gauge coupling g' is defined as

$$\frac{1}{g'^2} = \frac{1}{g_1'^2} + \frac{1}{g_2'^2}. \quad (3)$$

We can also express the coupling constants in terms of the electron charge e and the analog of the weak mixing angle θ_W in the SM as

$$g = \frac{e}{\sin \theta_W}, \quad g' = \frac{e}{\cos \theta_W}. \quad (4)$$

To break the $SU(2)_L \times U(1)_Y$ gauge symmetry down to the $U(1)_{EM}$, two scalar Higgs doublets $\Phi_{1,2}$ are introduced, which transform under $SU(2)_L \times U(1)_1 \times U(1)_2$ as $(\mathbf{2}, 1/2, 0)$ and $(\mathbf{2}, 0, 1/2)$, respectively, and develop VEVs as

$$\langle \Phi_{1,2} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{1,2} \end{pmatrix}. \quad (5)$$

Associated with this symmetry breaking, the W -boson gets mass $M_W^2 = g^2(v_1^2 + v_2^2)/4$ while the mass-squared matrix of the neutral gauge bosons is given by

$$M_{neutral}^2 = \frac{u^2}{4} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (g^2 + g'^2)\epsilon & \frac{g'^2 \epsilon}{\sin \theta_W} \cot \phi \cos^2 \beta (\tan^2 \phi - \tan^2 \beta) \\ 0 & \dots & \frac{4g'^2}{\sin^2 2\phi} + g'^2 \epsilon (\cos^2 \beta \tan^2 \phi + \sin^2 \beta \cot^2 \phi) \end{pmatrix}, \quad (6)$$

under the basis $(A_\mu, Z_\mu, \tilde{B}_\mu)$ with

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}, \quad (7)$$

and

$$\epsilon = \frac{v_1^2 + v_2^2}{u^2}, \quad \tan \beta \equiv \frac{v_2}{v_1}. \quad (8)$$

There is a mixing between Z_μ and \tilde{B}_μ which is constrained to be very small $\lesssim 10^{-3}$ by the electroweak precision measurements [13]. Thus we fix model-parameters to realize such a small mixing, so that the field \tilde{B}_μ is identified as the Z' -boson. Note that from Eq. (6), the mixing between Z_μ and \tilde{B}_μ vanishes for $\tan^2 \phi = \tan^2 \beta$. In the following, we take $\tan^2 \phi = \tan^2 \beta$, for simplicity.

Assigning $Y_2 = 0$ for fermions in the first two generations, while $Y_1 = 0$ for fermions in the third generation, we obtain the interactions between the Z' -boson and the SM fermions such as

$$-\mathcal{L}_{Z'} = \bar{\psi}_f \gamma^\mu (g_L^f P_L + g_R^f P_R) \psi_f Z'_\mu, \quad (9)$$

where $g_{L,R}$ for each SM fermion are given as

$$\begin{aligned}
g_L^{u,d,c,s} &= -\frac{1}{6} \frac{e}{\cos \theta_W} \tan \phi \quad , \quad g_L^{t,b} = \frac{1}{6} \frac{e}{\cos \theta_W} \cot \phi ; \\
g_L^{\nu_e, \nu_\mu, e, \mu} &= \frac{1}{2} \frac{e}{\cos \theta_W} \tan \phi \quad , \quad g_L^{\nu_\tau, \tau} = -\frac{1}{2} \frac{e}{\cos \theta_W} \cot \phi ; \\
g_R^{u,c} &= -\frac{2}{3} \frac{e}{\cos \theta_W} \tan \phi \quad , \quad g_R^t = \frac{2}{3} \frac{e}{\cos \theta_W} \cot \phi ; \\
g_R^{d,s} &= \frac{1}{3} \frac{e}{\cos \theta_W} \tan \phi \quad , \quad g_R^b = -\frac{1}{3} \frac{e}{\cos \theta_W} \cot \phi ; \\
g_R^{e,\mu} &= \frac{e}{\cos \theta_W} \tan \phi \quad , \quad g_R^\tau = -\frac{e}{\cos \theta_W} \cot \phi ; \\
g_R^{\nu_e, \nu_\mu, \nu_\tau} &= 0 \quad .
\end{aligned} \tag{10}$$

As a result, the Z' -boson couples differently to the first two and the third generations. In general, the family non-universal couplings generate tree-level flavor changing neutral currents (FCNCs) [12, 14, 15, 16] and therefore, they are severely constrained by current experimental data. The constraints on the model parameters due to the FCNC processes have been examined in Ref. [12], and in our analysis on the Z' -boson phenomenology, a parameter set is chosen to be consistent with the current experiments.

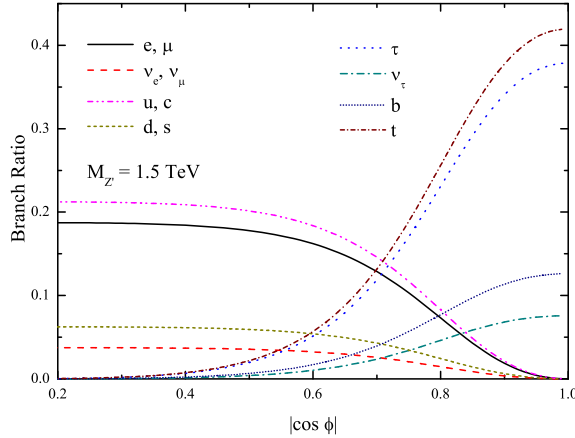


Figure 1: The branching ratios of the decay $Z' \rightarrow \bar{f}f$ as a function of $|\cos \phi|$ for $M_{Z'} = 1.5$ TeV .

The branching ratios of the decay $Z' \rightarrow \bar{f}f$ as a function of $|\cos \phi|$ are shown in Fig. 1. As can be expected, the branching ratios into the fermions in the first two generations are different from those into the corresponding fermions in the third generation (except for $\tan \phi = 1$). In the limit, $\cot \phi = 0$, the couplings between the Z' -boson and the third generation fermions are switched off, while the other limit, $\tan \phi = 0$, the couplings with the

first two generation fermions vanish. For $\tan \phi = 1$, the couplings of the third generation fermions becomes the same (up to sign) as those of the corresponding fermions in the first two generations. Since the sign difference does not appear in the Z' decay width, no flavor-dependence can be seen in Fig. 1 for $\tan \phi = 1$. However, as will be shown below, this sign difference causes significant differences at collider phenomenologies.

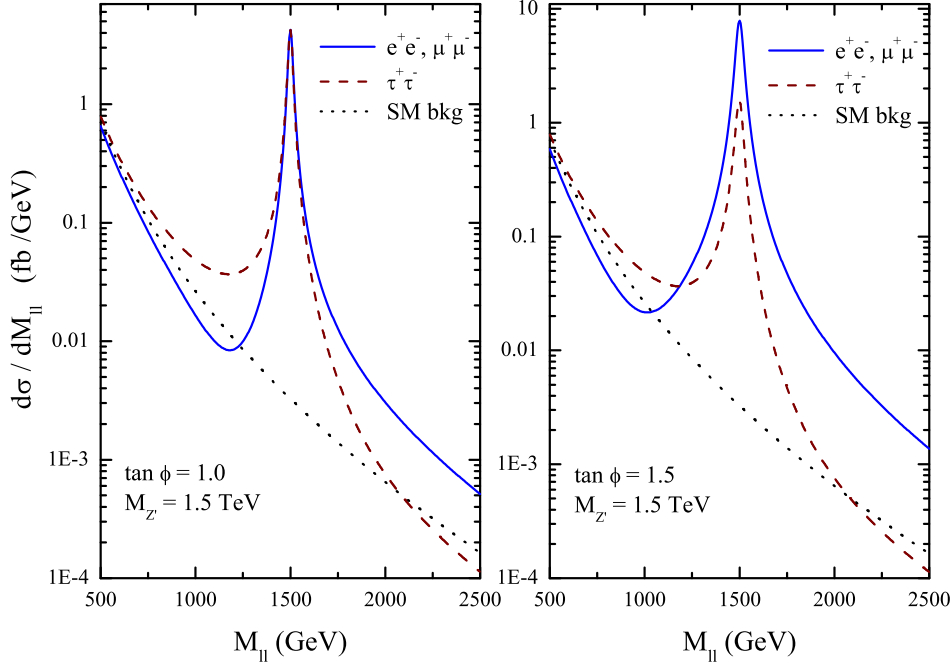


Figure 2: The differential cross section for $pp \rightarrow \ell^+\ell^-X$ at the LHC for $M_{Z'} = 1.5$ TeV and $\tan \phi = 1.0$ (left) and 1.5 (right), together with the SM cross section mediated by the Z -boson and photon (dotted line).

Now we investigate the Z' -boson production at the LHC. We calculate the dilepton production cross sections through the Z' exchange together with the SM processes mediated by the Z -boson and photon¹. The significance of the Z' -boson discovery through the process $pp \rightarrow \ell^+\ell^-X$ ($\ell^+\ell^- = e^+e^-, \mu^+\mu^-$) has been investigated in Ref. [12]. Here we show the

¹ The quark pair production channel, in particular, top-quark pair production via the Z' -boson exchange is also worth investigating [17], since top quark, which electroweakly decays before hadronization, can be used as an ideal tool to probe new physics beyond the Standard Model [18].

dependence of the cross section on the final state invariant mass M_{ll} described as²

$$\begin{aligned} \frac{d\sigma(pp \rightarrow \ell^+ \ell^- X)}{dM_{ll}} &= \sum_{a,b} \int_{-1}^1 d\cos\theta \int_{\frac{M_{ll}^2}{E_{\text{CMS}}^2}}^1 dx_1 \frac{2M_{ll}}{x_1 E_{\text{CMS}}^2} \\ &\times f_a(x_1, Q^2) f_b\left(\frac{M_{ll}^2}{x_1 E_{\text{CMS}}^2}, Q^2\right) \frac{d\sigma(\bar{q}q \rightarrow \ell^+ \ell^-)}{d\cos\theta}, \end{aligned} \quad (11)$$

where $E_{\text{CMS}} = 14$ TeV is the center-of-mass energy of the LHC. In our numerical analysis, we employ CTEQ5M [19] for the parton distribution functions with the factorization scale $Q = M_{Z'}$.

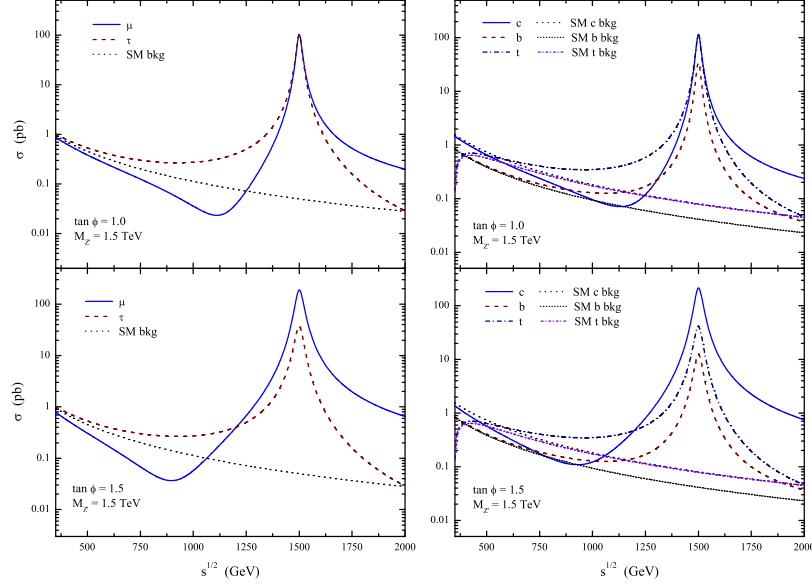


Figure 3: The cross sections for $e^+e^- \rightarrow \bar{f}f$ at the ILC as a function of the center-of-mass energy \sqrt{s} .

Fig. 2 shows the differential cross section for $pp \rightarrow e^+e^-(\mu^+\mu^-)$ and $\tau^+\tau^-$ for $M_{Z'} = 1.5$ TeV and $\tan\phi = 1.0$ (left) and 1.5 (right), together with the SM cross section mediated by the Z -boson and photon. Although for $\tan\phi = 1$ the peak cross sections are the same, the dependence of dilepton invariant mass shows a remarkable flavor-dependence. This is because for the cross sections away from the peak, the interference between the Z' -boson mediated process and the SM processes dominates and the sign difference of the couplings

² For explicit formulas for the production cross section etc, see, for example, Appendix in [17].

in Eq. (11) between the first two and third generation fermions causes the strong flavor-dependence of the cross sections. For $\tan \phi = 1.5$, the flavor-dependence appears even in the peak cross sections.

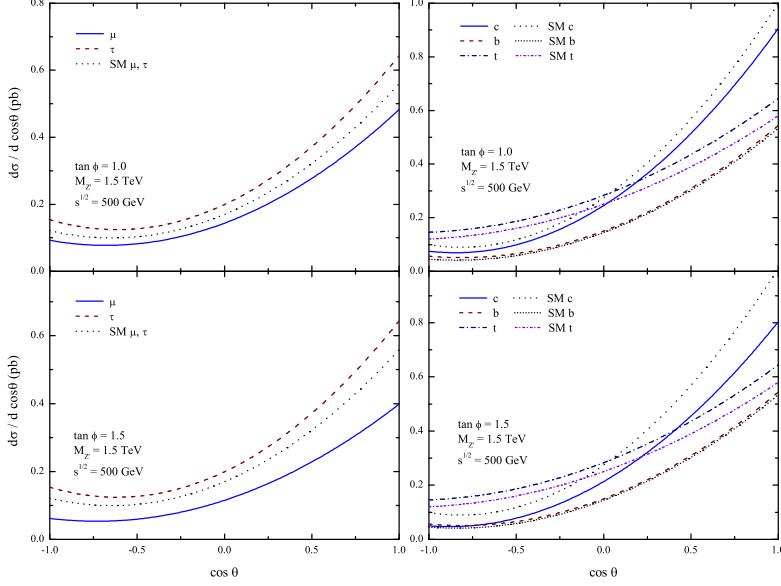


Figure 4: The differential cross section $d\sigma/d\cos\theta$ for the process $e^+e^- \rightarrow \bar{f}f$ at ILC with $\sqrt{s} = 500$ GeV.

When we choose a kinematical region for the invariant mass in the range, $M_{Z'} - \Gamma_{Z'} = 1.35$ TeV $\leq M_{ll} \leq M_{Z'} = 1.5$ TeV, for example, 7.8×10^3 and 8.8×10^3 signal events would be observed for e^+e^- ($\mu^+\mu^-$) and $\tau^+\tau^-$ channels, respectively, with an integrated luminosity of 100 fb^{-1} , while the number of events for the SM background would be about 100. In the case $\tan \phi = 1.5$, we would expect 4.7×10^4 and 9.4×10^3 signal events for e^+e^- ($\mu^+\mu^-$) and $\tau^+\tau^-$ channels, respectively, for the kinematical range around the peak, $M_{Z'} - \Gamma_{Z'} = 1.35$ TeV $\leq M_{ll} \leq M_{Z'} + \Gamma_{Z'} = 1.65$ TeV, with an integrated luminosity of 100 fb^{-1} .

Once a resonance of the Z' -boson has been discovered at the LHC, the Z' -boson mass can be determined from the peak energy of the dilepton invariant mass. The difference between the cross sections of different dilepton channels at the LHC could be a good distinction between flavor-dependent Z' models and the flavor-universal ones. The ILC can provide more precise measurement of the Z' -boson properties such as couplings with each (chiral) SM fermion, spin and etc., even if its center-of-mass energy is far below the Z' -boson mass [8]. Then, we next study ILC phenomenologies on Z' -boson.

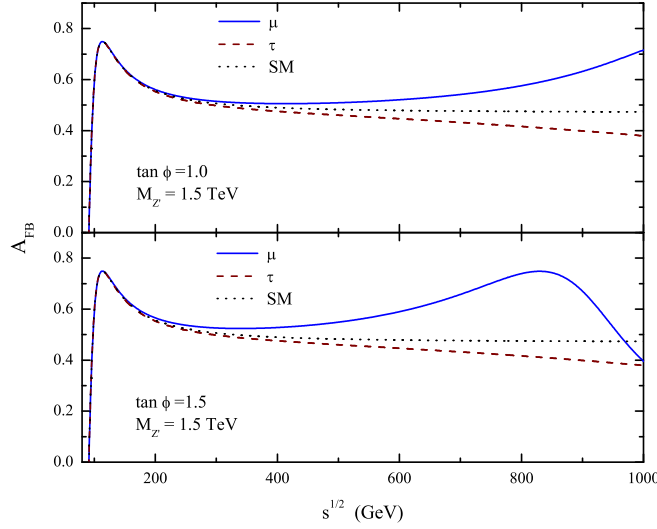


Figure 5: The forward-backward asymmetry A_{FB}^ℓ as a function of the center-of-mass energy \sqrt{s} .

We begin with calculating the cross sections of the process $e^+e^- \rightarrow \bar{f}f$ at the ILC with different energies and the results are depicted in Fig. 3 for $M_{Z'} = 1.5$ TeV and $\tan\phi = 1$ (upper figures) and 1.5 (lower figures), together with the SM cross sections. The results show a similar behavior as in Fig. 2 and we can see sizable deviations from the SM results and also clear flavor-dependences of the cross sections, even for $s \ll M_{Z'}^2$. For $\sqrt{s} = 500$ GeV fixed, we show the differential cross sections for the process $e^+e^- \rightarrow \bar{f}f$ for $\tan\phi = 1$ (upper figures) and 1.5 (lower figures) in Fig. 4. Since the collider energy is lower than $M_{Z'}$, the interference between the SM processes and the virtual Z' -boson exchange causes the deviations of the cross sections from the SM results. We can see that the deviations from the SM results are flavor-dependent. With high integrated luminosity, the deviations could be used to determine the Z' couplings and therefore identify the Z' -boson properties.

Furthermore, the forward-backward asymmetries A_{FB}^ℓ are very distinct for different generation dileptons³ as shown in Fig. 5. Here we have defined the forward-backward asymmetry for $e^+e^- \rightarrow \ell^+\ell^-$ ($\ell = \mu(e)$ or τ) as

$$A_{FB} \equiv \frac{\int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta}{\int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta + \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta} \quad (12)$$

with θ being the scattering angle between the final state ℓ^- and the initial e^- beam directions. The precise measurements of A_{FB}^ℓ at the ILC would reveal not only the existence of the Z' -boson at very high energy but also its flavor-dependent couplings.

³ Similar analysis on the model proposed in [11] was performed in [20].

In summary, we have studied the signatures of a new charge neutral gauge boson, Z' , at the LHC and ILC. Such a gauge boson has been predicted in many new physics models beyond the SM. In particular, we have concentrated on a class of these models where Z' -boson has flavor-dependent couplings with the SM fermions. As a concrete example of such models, we have taken the top hypercharge model proposed in [12], where the SM fermions in the third generation have couplings with the Z' -boson different from those of the corresponding SM fermions in the first two generations. For a Z' -boson mass around 1 TeV, the dilepton production cross sections via the Z' -boson in the s -channel well-exceed the SM background, so that the discovery of the Z' -boson resonance would be promising in this case. In addition, the dependence of the cross sections on the dilepton invariant mass shows a clear flavor-dependence around the resonance peak and therefore, the flavor-non-universality of the Z' -boson resonance could be also observed at the LHC. We have also analyzed the Z' -boson effects at the ILC. Even if the energy at the ILC is far below a Z' -boson mass, the differential cross sections and the forward-backward asymmetries for the fermion pair production processes show not only sizable deviations from the SM results but also significant flavor-dependences, through which the ILC with a high integrated luminosity could precisely measure the flavor-dependent couplings of the Z' -boson with the SM fermions in different generations. Finally, although the analysis in this Letter was based on the model proposed in [12], our strategy is applicable to general models of Z' -boson with flavor-dependent couplings.

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